CALCULATION OF HEAT TRANSFER AND HYDRAULIC RESISTANCE IN LAMINAR FLOW OF AN EQUILIBRIUM DISSOCIATING GAS IN A CIRCULAR TUBE

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Numerical solution of the boundary layer equations has been used to obtain the temperature and velocity fields, as well as the local heat transfer and friction coefficients, in laminar flow of equilibrium dissociating hydrogen in a circular tube.

A number of papers investigating the effect of dissociation on heat transfer and friction in tubes have considered flow either far from the tube entrance [1], or with an established velocity profile [2, 3], and, in addition, the fluid was considered incompressible. The present paper considers the problem of simultaneous development of the velocity and temperature profiles in the flow of an equilibrium dissociating compressible gas.

We consider established laminar flow of a compressible gas in a comparatively long circular tube, assuming that there is equilibrium dissociation throughout the whole flow volume. Then, as is known, the effect of the dissociation reaction can be evaluated by introducing the so-called effective physical properties which, along with molecular transport, account for the effects of transfer due to chemical reactions, and the determination of temperature and velocity profiles reduces to solution of a system of ordinary boundary-layer equations

$$\begin{gathered}
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r}\left(r\mu \frac{\partial u}{\partial r}\right) - \frac{\partial P}{\partial x};\\
g\rho u \frac{\partial h}{\partial x} + g\rho v \frac{\partial h}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda \frac{\partial T}{\partial r}\right) + u\frac{\partial P}{\partial x} + A\mu\left(\frac{\partial u}{\partial r}\right)^2;\\
\frac{\partial \rho u r}{\partial x} + \frac{\partial \rho v r}{\partial r} = 0;\\
\frac{\partial P}{\partial r} = 0;\\
\rho = \frac{Pm}{g(1+\alpha)RT},
\end{gathered} \tag{1}$$

in which the physical properties depend appreciably on temperature and pressure.

The following boundary conditions are considered:

$$\begin{gathered}
r_0 \geqslant r \geqslant 0, \quad x = 0, \quad T = T_0, \quad u = V_0, \quad P = P_0;\\
r = r_0, \quad x \geqslant 0, \quad T = T_1, \quad u = 0, \quad v = 0;\\
r = 0, \quad \frac{\partial u}{\partial r} = \frac{\partial T}{\partial r} = 0, \quad v = 0.
\end{gathered} \tag{2}$$

System (1) was solved simultaneously with (2), numerically on a 3M high-speed electronic computer. For this purpose system (1) was approximated using a two-layer implicit six-point scheme. A basic rectangular

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grid x = nΔx, R = mΔR was chosen in the x, R plane, where m, n = 0, 1, 2, ..., and an auxiliary grid x = (n + 1/2)Δx, R = mΔR.

Using dimensionless variables, system (1) is written in finite difference form as follows:

$$(\rho\bar{u})_m^{n-1/2} \frac{\bar{u}_m^n - \bar{u}_m^{n-1}}{\Delta x} + 2(\rho\bar{v})_m^{n-1/2} \frac{s_1(\bar{u}_{m+1}^n - \bar{u}_{m-1}^n) + (1-s_1)(\bar{u}_{m+1}^{n-1} - \bar{u}_{m-1}^{n-1})}{2\Delta R}$$

$$= \frac{\delta}{\Delta R^2}\Big\{\left[(1-s_1)(\bar{u}_{m+1}^{n-1} - \bar{u}_m^{n-1}) + s_1(\bar{u}_{m+1}^n - \bar{u}_m^n)\right]\frac{\mu_{m+1}^{n-1/2} + \mu_m^{n-1/2}}{2}$$

$$- \left[(1-s_1)(\bar{u}_m^{n-1} - \bar{u}_{m-1}^{n-1}) + s_1(\bar{u}_m^n - \bar{u}_{m-1}^n)\right]\frac{\mu_m^{n-1/2} + \mu_{m-1}^{n-1/2}}{2}\Big\}$$

$$+ \delta\left(\frac{\mu}{R}\right)_m^{n-1/2} \frac{s_1(\bar{u}_{m+1}^n - \bar{u}_{m-1}^n) + (1-s_1)(\bar{u}_{m+1}^{n-1} - \bar{u}_{m-1}^{n-1})}{2\Delta R} - \frac{\partial \bar{P}}{\partial \bar{x}};$$

$$(a\bar{u})_m^{n-1/2} \frac{\Theta_m^n - \Theta_m^{n-1}}{\Delta x} + (2a\bar{v})_m^{n-1/2} \frac{s_2(\Theta_{m+1}^n - \Theta_{m-1}^n) + (1-s_2)(\Theta_{m+1}^{n-1} - \Theta_{m-1}^{n-1})}{2\Delta R}$$

$$= \frac{\beta}{\Delta R^2}\Big\{\left[(1-s_2)(\Theta_{m+1}^{n-1} - \Theta_m^{n-1}) + s_2(\Theta_{m+1}^n - \Theta_m^n)\right]\frac{\lambda_{m+1}^{n-1/2} + \lambda_m^{n-1/2}}{2}$$

$$- \left[(1-s_2)(\Theta_m^{n-1} - \Theta_{m-1}^{n-1}) + s_2(\Theta_m^n - \Theta_{m-1}^n)\right]\frac{\lambda_m^{n-1/2} + \lambda_{m-1}^{n-1/2}}{2}\Big\}$$

$$+ \beta\left(\frac{\lambda}{R}\right)_m^{n-1/2} \frac{s_2(\Theta_{m+1}^n - \Theta_{m-1}^n) + (1-s_2)(\Theta_{m+1}^{n-1} - \Theta_{m-1}^{n-1})}{2\Delta R} + \vartheta\frac{\partial \bar{P}}{\partial \bar{x}}\left[(1-s_1)\bar{u}_m^{n-1} + s_1\bar{u}_m^n\right]$$

$$+ C\mu_m^{n-1/2} \frac{\left[s_1(\bar{u}_{m+1}^n - \bar{u}_{m-1}^n) + (1-s_1)(\bar{u}_{m+1}^{n+1} - \bar{u}_{m+1}^{n-1})\right]^2}{4\Delta R^2}; \tag{3}$$

$$\frac{1}{2\Delta x}\left[(\rho\bar{u}R)_m^n - (\rho\bar{u}R)_m^{n-1} + (\rho\bar{u}R)_{m+1}^n - (\rho\bar{u}R)_{m+1}^{n-1}\right] + \frac{2}{\Delta R}\left[(\rho\bar{v}R)_{m+1}^{n-1/2} - (\rho\bar{v}R)_m^{n-1/2}\right] = 0,$$

where

$$\bar{x} = x/d;\quad \bar{u} = u/V_0;\quad \Theta = (T - T_1)/(T_0 - T_1);\quad a = g\rho c_p;\quad \bar{P} = P/V_0^2;\quad \bar{v} = v/V_0;$$

$$\beta = 4/3600 dV_0;\quad \vartheta = AV_0^2/(T_0 - T_1);\quad C = 4AV_0/d(T_0 - T_1);\quad \delta = 4/V_0 d.$$

The system of algebraic equations obtained was solved by the marching method [4, 5]. Since the solution requires knowledge of the coefficients $(\rho\bar{u})_m^{n-1/2}$, $(\rho\bar{v})_m^{n-1/2}$, etc., at the center of the computing layer, the following iteration process was constructed. First the energy equation was solved in the zero approximation, in which all the unknown coefficients were given the corresponding values in the preceding layer, and then the equations of motion and continuity were solved similarly; thereafter the first approximation calculation was performed, in which the unknown coefficients were determined from the results of the zero approximation.

The iteration process was stopped when the temperature and velocity profiles of the last approximation did not differ by more than a given amount from the previous approximation. The additional condition

$$\int_0^1 \rho\bar{u}R\,dR = \text{const}$$

was used to determine the pressure gradient in the equation of motion.

The physical properties were given in tabular form. Here it was assumed that the pressure variation along the tube had a slight effect on the effective properties, which were given at the pressure P_0. In the calculations the step sizes Δx and ΔR were reduced until the result did not differ from the previous result in the third place. The heat-transfer coefficient was calculated from the following formula:

$$\mathrm{Nu} = \frac{2\left.\dfrac{\partial\Theta}{\partial R}\right|_{R=1}}{\Theta_w - \Theta_{av}}, \tag{4}$$

where Φ_{av} is the mean temperature of the gas, determined from the average calorimetric enthalpy.

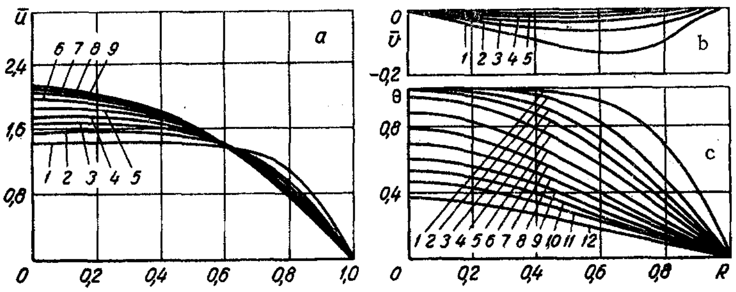

Fig. 1

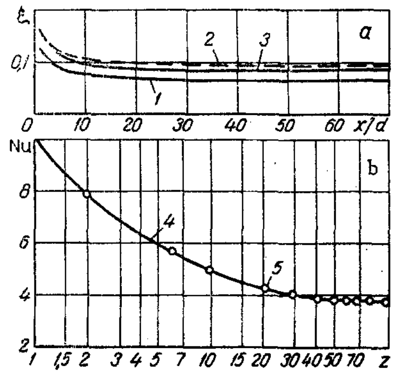

Fig. 2

Fig. 1. Distribution of dimensionless profiles of velocity and temperature across the tube section as a function of x/d (Re = 744; T_0 = 681°K; T_1 = 700°K; Δx = 0.025; ΔR = 0.05): 1) x/d = 1; 2) 3; 3) 5; 4) 7; 5) 10; 6) 15; 7) 20; 8) 25; 9) 30; 10) 35; 11) 40; 12) 50; a) axial velocity component; b) radial velocity component; c) temperature.

Fig. 2. Variation of drag coefficient ξ as a function of x/d and of Nu as a function of z (Re = 744; T_0 = 681°K; T_1 = 700°K; ΔR = 0.05; Δx = 0.025): 1) calculated from Eq. (5); 2) from the formula $\xi = -2(d\bar{P}/d\bar{x})/\rho\bar{u}_0^2$; 3) from the formula of [7]; 4) the data of [6]; 5) from Eq. (4).

The hydraulic resistance coefficient was calculated from the formula

$$\xi = -\frac{2\dfrac{\partial}{\partial \bar{x}}\left(\overline{\rho u^2_{av}} + \bar{P}\right)}{\overline{\rho u^2_{av}}}, \tag{5}$$

where

$$\overline{\rho u^2_{av}} = \int_0^1 2\bar{\rho}\bar{u}^2 R dR.$$

To check the correctness of the method, calculations were performed for small temperature differences between the wall and the gas, which allowed the results to be compared with others obtained for constant physical properties. Figure 1 shows the distribution of dimensionless gas temperature and of velocity components across the tube section for various values of x/d. Figure 2 shows results of calculation of Nu and the drag coefficient. Data of [6, 7] are given for comparison.

Examination of these figures shows that the value of Nu agrees with that obtained for flow with a parabolic velocity profile [6], while the drag coefficient, depending on its definition, differs somewhat from the results of the approximate solution of Targ, given in [7], associated with change of the mean velocity along the tube (in [7] the density and the mean velocity were constant).

In order to elucidate the effect of equilibrium dissociation on heat transfer and friction, a series of calculations was made with various values of the initial parameters. Since the effective properties of a dissociating gas, and of hydrogen, in particular, typically have extrema in their variation with temperature, the following three characteristic ranges of variation of the degree of dissociation in the boundary layer can be identified: 1) α = 0-1; 2) α = 0-0.5; 3) α = 0.5-1.

Additionally, for similar conditions at the entrance, calculations were performed for nondissociating hydrogen with frozen composition, independent of temperature. To exclude the effect of compressibility on heat transfer, a mass flux was chosen, corresponding to a value of M at the tube entrance of ~0.01.

The results of the computations show that the equilibrium dissociation had the greatest effect on temperature profiles. Figure 3 shows dimensionless temperature profiles, calculated for the case of heating of equilibrium dissociated and "frozen" hydrogen. Examination of Fig. 3 shows that, depending on the degree of dissociation in the boundary layer, the temperature profile, particularly near the wall, can differ appreciably from the temperature profile in frozen flow of hydrogen. The temperature profiles also differ appreciably in the case of cooling of equilibrium dissociated hydrogen.

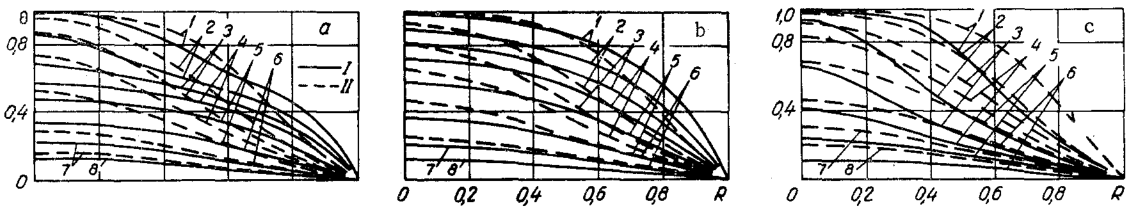

Fig. 3. Distribution of dimensionless temperature profile across the tube section, as a function of x/d, for equilibrium dissociated and "frozen" hydrogen (P_0 = 5 atm): 1) x/d = 1; 2) 3; 3) 5; 4) 7; 5) 10; 6) 15; 7) 20; 8) 25; a) Re = 424; α_1 = 1; T_0 = 2000°K; T_1 = 7000°K; b) Re = 235; α_0 = 0.334; α_1 = 1; T_0 = 4000°K; T_1 = 7000°K; c) Re = 424; α_0 = 0; α_1 = 0.334; T_0 = 2000°K; T_1 = 4000°K; I) equilibrium dissociated hydrogen; II) "frozen" hydrogen.

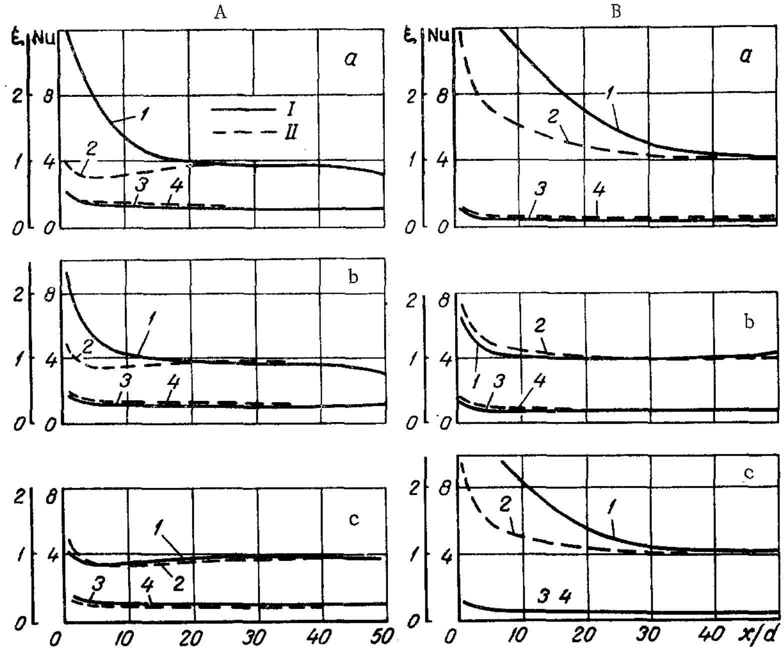

Fig. 4. Distributions of Nu and of the drag coefficient ξ along the tube for heating (A) and cooling (B) of hydrogen (P_0 = 5 atm): A: a) Re = 424; α_0 = 0; α_1 = 1; T_0 = 2000°K; T_1 = 7000°K; b) Re = 235; α_0 = 0.334; α_1 = 1; T_0 = 4000°K; T_1 = 7000°K; c) Re = 424; α_0 = 0; α_1 = 0.334; T_0 = 2000°K; T_1 = 4000°K; B: a) Re = 183; α_0 = 1; α_1 = 0; T_0 = 7000°K; T_1 = 2000°K; b) Re = 183; α_0 = 1; α_1 = 0.33; T_0 = 7000°K; T_1 = 4000°K; c) Re = 235; α_0 = 0.334; α_1 = 0; T_0 = 4000°K; T_1 = 2000°K; I, II) see Fig. 3; 1) Nu; 2) Nu_0; 3) ξ; 4) ξ_0.

Figure 4 shows Nu and the drag coefficients, calculated from Eqs. (4) and (5), for heating and cooling of equilibrium dissociated and "frozen" hydrogen. The analysis made of the results obtained has shown that the Nu value for equilibrium dissociation increases, in comparison with the value Nu_0 for frozen flow, as the degree of dissociation changes in the boundary layer in the range 0 to 1 or from 0.5 to 1 in the case of heating, and in the range 1 to 0 or 0.5 to 0 in the case of cooling. There is a decrease of Nu in comparison with Nu_0 with change of the degree of dissociation in the boundary layer from 0 to 0.5 on heating, and from 0.5 to 1 on cooling of the gas.

This effect on the Nu number is connected with the marked change of the effective physical properties in the boundary layer. It should be noted that the heat flux to the tube wall in all the cases examined is considerably greater than the heat flux in frozen flow, a feature which is connected with the appreciably greater values, averaged over the tube cross section, of effective heat capacity and thermal conductivity of the dissociated gas.

The velocity profiles are affected much less by the equilibrium dissociation, and so the maximum difference between the friction coefficients in equilibrium dissociation in frozen flow of hydrogen, ξ and ξ_0 does not exceed 10% for the above conditions.

NOTATION

u, v	are the velocity components;
x, r	are the coordinates;
V_0	is the gas velocity at the tube entrance;
T_1, T_0	are the temperatures of the wall and the gas at the tube entrance;
d	is the tube diameter;
P_0	is the pressure at the tube entrance;
ρ	is the density;
α_1, α_2	are the degree of dissociation at temperatures T_1 and T_0;
$R = r/r_0$;	
$z = x \cdot 10^3/(dPrRe)$;	
A	is the thermal equivalent of mechanical work;
M	is the Mach number;
Re	is the Reynolds number;
Nu	is the Nusselt number;
Nu_0	is the Nusselt number in frozen flow;
s_1, s_2	are the averaging parameters.

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